Poor man's scaling approach: Examples

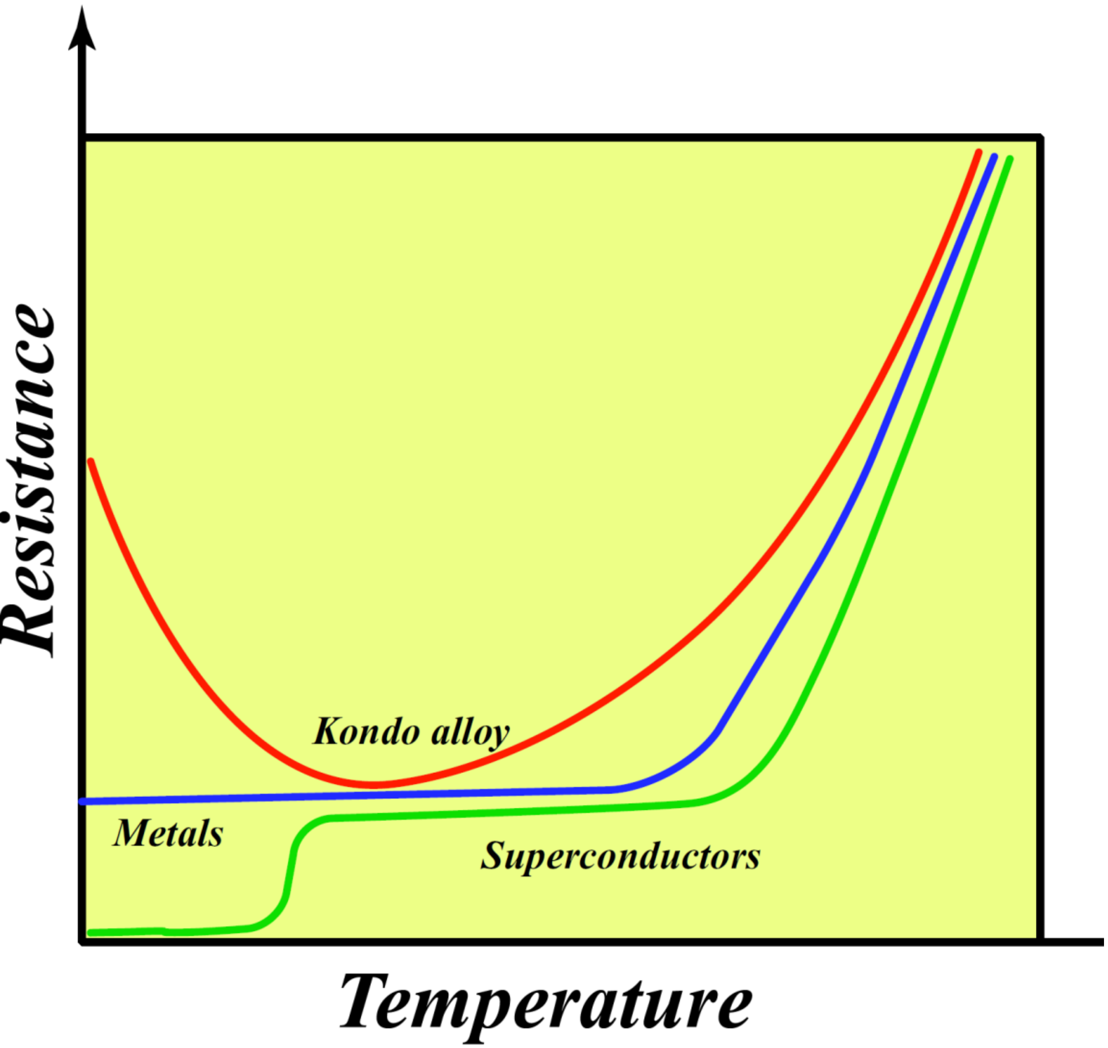
Mikhail Katsnelson

Outline

1. Original idea (P. W. Anderson, 1970): Kondo problem for a single magnetic impurity
2. Anisotropic Kondo problem (scattering by tunneling centers in metallic glasses
3. Infrared divergences: scattering by crystal-field-split centers
4. Infrared divergences: scattering by local phonons
5. Kondo lattices: Interplay of Kondo effect and spin dynamics

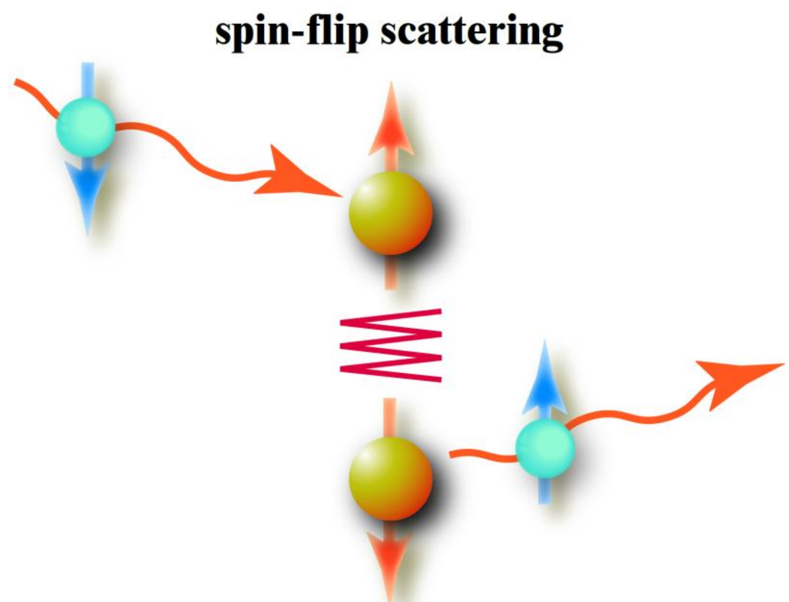
The main collaborator: Valya Irkhin, Institute of Metal Physics, Ekaterinburg

Single-impurity Kondo problem I



Resistivity minimum in noble metals at low T (1930^{th})

Attributed to magnetic impurities (Kondo 1964)

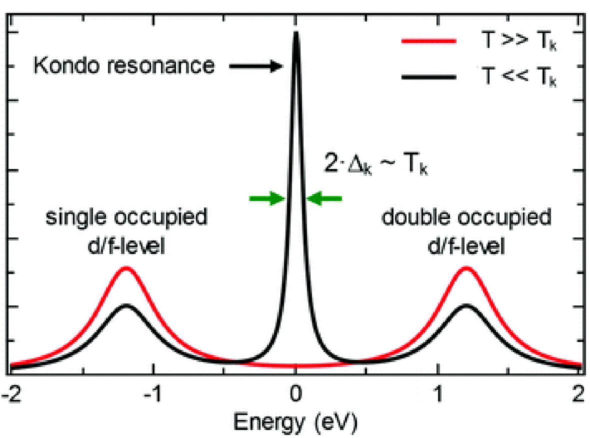


Single-impurity Kondo problem II

The simplest model: s-d exchange model (Vonsovsky 1946)

$$\hat{H} = \sum_{ij\sigma} t_{ij} \hat{c}^{+}_{i\sigma} \hat{c}_{j\sigma} - I \sum_{i} \hat{\vec{S}}_{i} \hat{\vec{s}}_{i}, \qquad \hat{\vec{s}}_{i} = \frac{1}{2} \sum_{\sigma\sigma'} \hat{c}^{+}_{i\sigma} \hat{\vec{\sigma}}_{\sigma\sigma'} \hat{c}_{i\sigma'}$$

At AFM I ($I < 0$) for one impurity: perturbation theory in I *is* divergent (Kondo 1964); a formation of Suhl-Abrikosov resonance (1965). At low temperatures: singlet ground state (Anderson – Hamann – Yuval 1970 and local Fermi liquid theory (Nozieres 1974)



To be specific; this is valid for $S = n/2$ (n number of scattering channels); for $S > n/2$ undercompensated regime ($S \to S - n/2$) For $S < n/2$ overcompensated regime and NFL behavior (Blandin and Nozieres, 1980)

Exact solution (Wiegmann 1980; Andrei 1980, and further work)

Poor man's scaling: Idea

A poor man's derivation of scaling laws for the Kondo problem

To cite this article: P W Anderson 1970 *J. Phys. C: Solid State Phys.* **3** 2436

A single-site scattering theory:

$$T(\omega) = V_{\text{int}} + V_{\text{int}} G_0(\omega) T(\omega)$$

Interaction: anisotropic s-d exchange coupling

$$V_{\text{int}} = \frac{J_\pm}{2}(S_+ s_- + S_- s_+) + J_z S_z s_z$$

Truncation of unperturbed Hamiltonian and thus base Green's function

$$G_0 = (\omega - \mathscr{H}_0)^{-1}$$

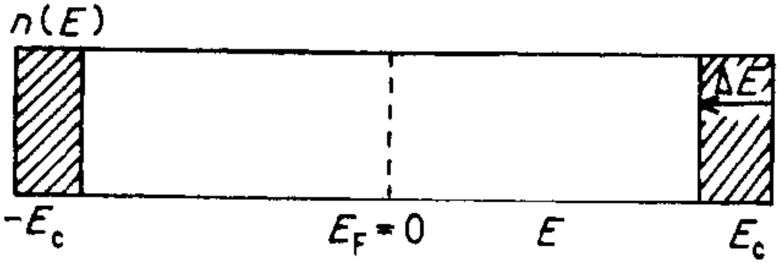
$$\mathscr{H}_0 = \sum_{\epsilon_k = E_F - E_c}^{E_F + E_c} \epsilon_k n_{k\sigma}$$

Exact Green's function and T-matrix:

$$G = G_0 + G_0 T G_0$$

Poor man's scaling: Idea II

Changing cutoff E_c



E_c to $E_c - \Delta E$, $\Delta E \ll E_c$

$E_c > \epsilon_{k''} > E_c - \Delta E$ projection operator $P_{\Delta E}$

projects *on to states containing one or more such particles*

$$T = V + VP_{\Delta E}G_0T + V(1 - P_{\Delta E})\,G_0T$$

$$T = V' + V'(1 - P_{\Delta E})\,G_0T$$

$$V' = V + VP_{\Delta E}G_0V$$

(approximate, small V) $\quad \mathrm{d}V = VP_{\Delta E}G_0V$

Poor man's scaling: Idea III

For our model (anisotropic s-d exchange model):

$$dV = \sum_{k_1,\sigma_1}^{|\epsilon_{k_1}|<E_c-\Delta E} \sum_{k_2,\sigma_2}^{|\epsilon_{k_2}|<E_c-\Delta E} \sum_{k,\sigma}^{E_c>|\epsilon_k|>E_c-\Delta E} \frac{1}{\omega - E_c - |\epsilon_{k_1}|}$$
$$\times \Big[(C^+_{k_2\sigma_2} C_{k\sigma})(C^+_{k\sigma} C_{k_1\sigma_1}) \left\{ \frac{J_\pm}{2} (S_+(s_-)_{\sigma_2\sigma} + S_-(s_+)_{\sigma_2\sigma}) + J_z S_z (s_z)_{\sigma_2\sigma} \right\}$$
$$\times \left\{ \frac{J_\pm}{2} (S_+(s_-)_{\sigma\sigma_1} + S_-(s_+)_{\sigma\sigma_1}) + J_z S_z (s_z)_{\sigma\sigma_1} \right\} + C^+_{k\sigma} C_{k_2\sigma_2} C^+_{k_1\sigma_1} C_{k\sigma} \left\{ \frac{J_\pm}{2} (S_+(s_-)_{\sigma\sigma_2} \right.$$
$$\left. + S_-(s_+)_{\sigma\sigma_2}) + J_z S_z (s_z)_{\sigma\sigma_2} \right\} \left\{ \frac{J_\pm}{2} (S_+(s_-)_{\sigma_1\sigma} + S_-(s_+)_{\sigma_1\sigma}) + J_z S_z (s_z)_{\sigma_1\sigma} \right\} \Big].$$

Simplifications for $S = ½$:

$$dV = \sum_{k_1,\sigma_1} \sum_{k_2,\sigma_2} \frac{\rho \Delta E}{\omega - E_c - |\epsilon_{k_1}|} \Big[C^+_{k_2\sigma_2} C_{k_1\sigma_1} \Big\{ \delta_{\sigma_1\sigma_2} \left(\frac{J_\pm^2}{8} + \frac{J_z^2}{16} \right) - \frac{J_\pm^2 S_z (s_z)_{\sigma_2\sigma_1}}{2}$$
$$- \frac{J_\pm J_z}{4} (S_+(s_-)_{\sigma_2\sigma_1} + S_-(s_+)_{\sigma_2\sigma_1}) \Big\} + C_{k_2\sigma_2} C^+_{k_1\sigma_1} \Big\{ \delta_{\sigma_1\sigma_2} \left(\frac{J_\pm^2}{8} + \frac{J_z^2}{16} \right) + \frac{J_\pm^2 S_z (s_z)_{\sigma_1\sigma_2}}{2}$$
$$+ \frac{J_\pm J_z}{4} (S_+(s_-)_{\sigma_1\sigma_2}) \Big\} \Big]. \tag{10}$$

Poor man's scaling: Idea IV

Shift of the ground-state energy

$$\Delta(E_g) = \int_{E_c}^{E_c^0} \mathrm{d}E \left(\frac{(J_z\rho)^2}{8} + \frac{(J_\pm\rho)^2}{4}\right) \ln\left(\frac{2E-\omega-\Delta}{E-\omega-\Delta}\right) \simeq \ln 2 \int_{E_c}^{E_c^0} \left(\frac{(J_z\rho)^2}{8} + \frac{(J_\pm\rho)^2}{4}\right) \mathrm{d}E.$$

$$\frac{\mathrm{d}V}{\Delta E} = \frac{\rho}{\omega - E_c + \Delta(E_c)} \sum_{k_1\sigma_1 k_2\sigma_2} C^+_{k_2\sigma_2} C_{k_1\sigma_1} \Big\{ -J_\pm^2 S_z(s_z)_{\sigma_2\sigma_1} - \frac{J_\pm J_z}{2}\left(S_+(s_-)_{\sigma_2\sigma_1} + S_-(s_+)_{\sigma_2\sigma_1}\right)\Big\}.$$

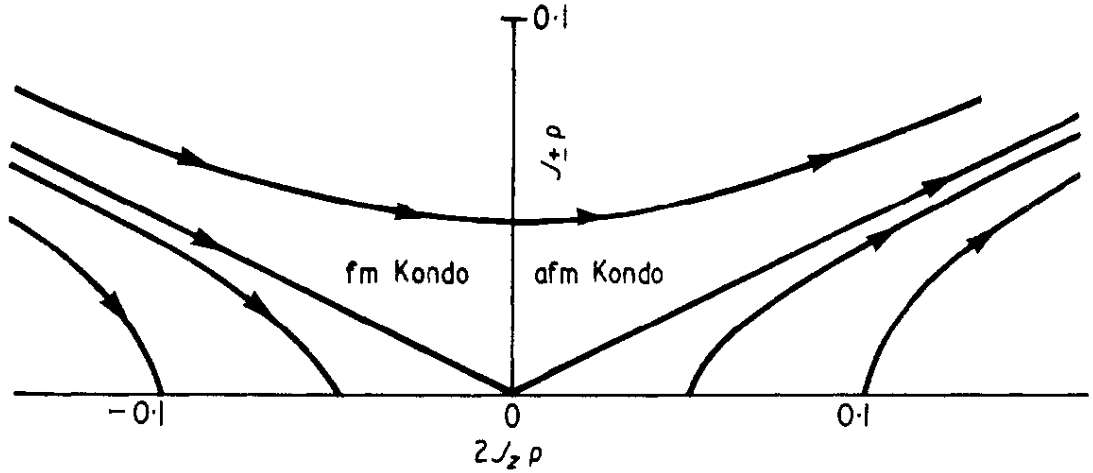
Pass to the differential equations:

$$\frac{\mathrm{d}J_z}{\mathrm{d}E_c} = -\frac{\rho}{\omega - E_c + \Delta} J_\pm^2$$

$$\frac{\mathrm{d}J_\pm}{\mathrm{d}E_c} = -\frac{\rho}{\omega - E_c + \Delta} J_z J_\pm$$

Poor man's scaling: Idea V

Solution of the equations $J_z^2 - J_\pm^2 = \text{const}$



FM ($J > 0$): effective coupling constant tends to zero, nothing interesting

AFM ($J < 0$): effective coupling constant *diverges* at

$$\omega_0 \simeq -\Delta(0) - E_c^0 \exp(-1/\rho J_0) = -\Delta(0) - E_K$$

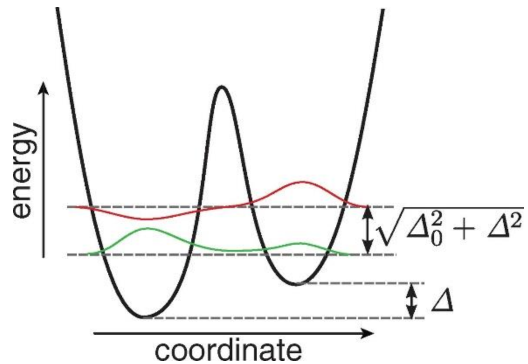
We cannot say what happens in the strong-coupling regime but we can find "Kondo energy", i.e. the border of strong-coupling regime

Two-level states in metallic glasses

Scaling consideration of a generalized anisotropic s–d exchange model for the interaction of electrons with two-level systems

Physics Letters A 213 (1996) 65–68

V.Yu. Irkhin [a], M.I. Katsnelson [a], A.V. Trefilov [b]

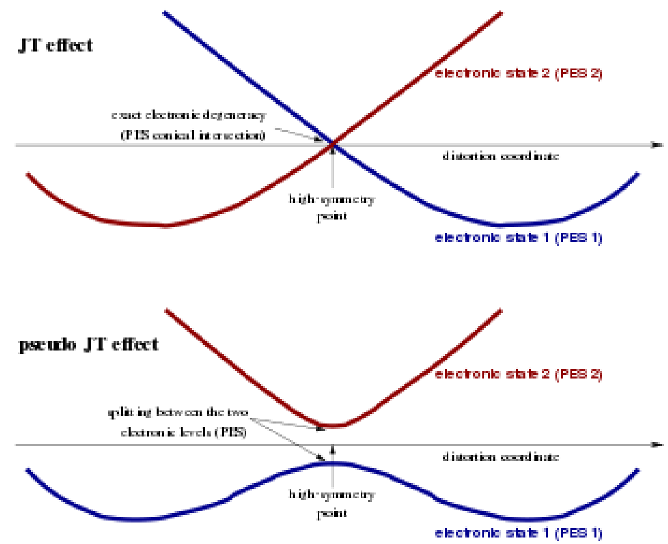


Atoms in double-well potentials in metallic glasses or highly anharmonic crystals

$$H = \sum_{k\tau} E_{k\tau} c^+_{k\tau} c_{k\tau} - \sum_{kq\tau\tau'} I_{\alpha\beta} S^\beta_q \sigma^\alpha_{\tau\tau'} c^+_{k\tau} c_{k-q,\tau'} \qquad (1)$$

where $c^+_{k\tau}$ are the creation operators for the conduction electrons with quasimomentum $\boldsymbol{k}$ and pseudospin projection $\tau = \pm$, the energy $E_{k\tau}$ is referred to E_F. The interaction Hamiltonian is taken for simplicity in the contact form, σ are the Pauli matrices, $I_{\alpha\beta}$ is the coupling parameter matrix. Fol-

Spin up – atom left, spin down – atom right



Crossing bands (dynamic Jahn-Teller effect)

Two-level states in metallic glasses II

General equations of poor man's scaling $\frac{\partial g^{\mathrm{ef}}_{\nu\mu}}{\partial \xi} = -\epsilon_{\nu\alpha\gamma}\,\epsilon_{\mu\beta\lambda}\,g^{\mathrm{ef}}_{\alpha\beta}\,g^{\mathrm{ef}}_{\gamma\lambda}$

$\xi = \ln|W/E|$ W is the cutoff parameter $g^{\mathrm{ef}}_{\alpha\beta} = \rho I^{\mathrm{ef}}_{\alpha\beta}$

ρ being the electron density of states at E_{F}

$|I_{z\beta}| \gg |I_{x\beta}|, |I_{y\beta}| \sim \epsilon$ where ϵ is an overlap parameter which is exponentially small in the barrier height factor.

Solution of the equations $\mathrm{d}\eta = 2\,g^{\mathrm{ef}}_{yy}\,\mathrm{d}\xi$ To be specific, $g_{yy} \geq 0$

$$g^{\mathrm{ef}}_{xx} = g_{xx}\cosh\eta - g_{zz}\sinh\eta,$$

$$g^{\mathrm{ef}}_{zz} = g_{zz}\cosh\eta - g_{xx}\sinh\eta,$$

$$g^{\mathrm{ef}}_{xx} = g_{xz}\cosh\eta + g_{zx}\sinh\eta,$$

$$g^{\mathrm{ef}}_{zx} = g_{zx}\cosh\eta + g_{xz}\sinh\eta,$$

Two-level states in metallic glasses III

$$\left(g_{yy}^{\text{ef}}\right)^2 = g_{yy}^2 + A \sinh 2\eta + \tfrac{1}{8}\alpha^2(\cosh 2\eta - 1)$$

Total cross section $\displaystyle\sum_{\alpha\beta} |\, g_{\alpha\beta}^{\text{ef}}(\xi)\,|^2 = \text{const} + 3\,|\, g_{yy}^{\text{ef}}(\xi)\,|^2$

$$A = g_{xz}g_{zx} - g_{xx}g_{zz},$$

$$\alpha = 2\left(g_{xx}^2 + g_{zz}^2 + g_{xz}^2 + g_{zx}^2\right)^{1/2}$$

$$= 2\left(g_{xx}^2 + g_{zz}^2\right)^{1/2} + \mathrm{O}(\epsilon).$$

$$\xi = \tfrac{1}{2}\int_0^{\eta} \frac{\mathrm{d}\eta'}{|\, g_{yy}^{\text{ef}}(\eta')\,|}$$

"Kondo temperature" (strong-coupling region) is determined by the condition

$$\xi(\eta \to \infty) = \ln(W/T_{\mathrm{K}})$$

Two-level states in metallic glasses IV

$$|I_{x\beta}|, |I_{y\beta}| \ll |I_{z\beta}|$$

Allows to solve eqs. explicitly

$$T_K = W\left(\tfrac{1}{4}\gamma\right)^{1/\alpha}$$

Resistivity

$$\Delta\rho(T) \sim f(T)$$

$$= \frac{\varphi^2(T) + 4\varphi(T)\left[1+\varphi^2(T)\right] A/\alpha^2}{\left[1-\varphi^2(T)\right]^2}$$

$$\varphi(T) = (T_K/T)^{\alpha}$$

Contrary to simple Kondo problem can be nonmonotonous

$$\cosh\eta = \frac{1+\frac{1}{16}\gamma^2\exp(2\alpha\xi)}{1-\frac{1}{16}\gamma^2\exp(2\alpha\xi)}$$

$$\sinh\eta = \frac{\frac{1}{2}\gamma\exp(\alpha\xi)}{1-\frac{1}{16}\gamma^2\exp(2\alpha\xi)}$$

where

$$\gamma = \left|2\frac{|g_{yy}|}{\alpha} + \frac{A}{\alpha^2}\right| \sim \epsilon.$$

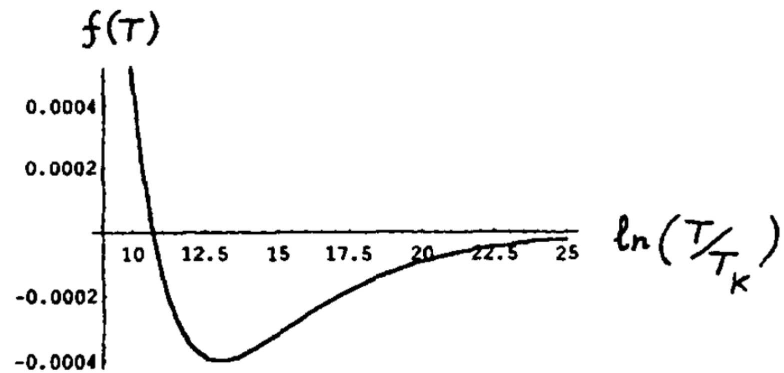


Fig. 1. $f(T)$ versus $\log(T/T_K)$ for $\alpha = 0.3$, $A/\alpha^2 = -0.01$.

$$T_{min} = T_K\left(\frac{\alpha^2}{2|A|}\right)^{1/\alpha} = \left|\frac{1}{8} - \frac{\alpha}{4|A|}|g_{yy}|\right|^{1/\alpha} W$$

Interaction with local excitations

Interaction of Conduction Electrons with Local Excitations
The Infrared Divergencies

Z. Phys. B – Condensed Matter 70, 371–378 (1988)

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Crystal-Field Splitting excitations
(e.g. 4f elements)

2. The Interaction with Local Pseudospin: The Singularities at $E \to E_F \pm \Delta$

Nondegenerate electron bands

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}}^{+} c_{\mathbf{k}} - \Delta s^z + \mathbf{J} \cdot \mathbf{s} c^{+} c$$

s^{α} are the 1/2-pseudospin operators

Second-order perturbation results for the electron self-energy

$$\delta \Sigma^{(2)}(E) = \frac{1}{2} \langle s^z \rangle J^+ J^- \sum_{\mathbf{q}} \left(\frac{f_{\mathbf{q}}}{E - \varepsilon_{\mathbf{q}} + \Delta} - \frac{f_{\mathbf{q}}}{E - \varepsilon_{\mathbf{q}} - \Delta} \right)$$

$$= \frac{1}{2} \langle s^z \rangle J^+ J^- \rho \ln \left| \frac{\varepsilon - \Delta}{\varepsilon + \Delta} \right| \qquad (2)$$

where $f_{\mathbf{q}} = f(\varepsilon_{\mathbf{q}})$ is the Fermi distribution function, ρ is the density of states on the Fermi level,

$$2 \langle s^z \rangle = n_+ - n_- = \tanh \frac{\Delta}{2T} \approx 1 \qquad (\Delta \gg T)$$

Interaction with local excitations II

$$G_{\mathbf{k}\mathbf{k}'}(E)=\langle\langle c_{\mathbf{k}}|c^{+}_{\mathbf{k}'}\rangle\rangle_E=-i\int_{-\infty}^{0}\mathrm{d}t\,\mathrm{e}^{-\mathrm{i}Et}\langle\{c_{\mathbf{k}},c^{+}_{\mathbf{k}'}(t)\}\rangle_e$$

Average over electron states but not on pseudospin – operator in pseudospin states!

$$G_{\mathbf{k}\mathbf{k}'}(E)=\frac{\delta_{\mathbf{k}\mathbf{k}'}}{E-\varepsilon_{\mathbf{k}}}+\frac{1}{E-\varepsilon_{\mathbf{k}}}T_{\mathbf{k}\mathbf{k}'}(E)\frac{1}{E-\varepsilon_{\mathbf{k}'}}$$

$$T(E)=\mathbf{J}_{\mathrm{ef}}(E)\,\mathbf{s}=\tfrac{1}{2}(J^{-}_{\mathrm{ef}}s^{+}+J^{+}_{\mathrm{ef}}s^{-})+J^{z}_{\mathrm{ef}}s^{z}$$

Separating log divergences in perturbation expansion at $\xi=\ln|\varDelta/(\varepsilon\mp\varDelta)|\to+\infty$

$$(E-\varepsilon_{\mathbf{k}})\,G_{\mathbf{k}\mathbf{k}'}(E)=\delta_{\mathbf{k}\mathbf{k}'}+\langle\langle(\mathbf{J}\cdot\mathbf{s})\,c|c^{+}_{\mathbf{k}'}\rangle\rangle_E,$$

$$\{\langle\langle(\mathbf{J}\cdot\mathbf{s})\,c|c^{+}_{\mathbf{k}'}\rangle\rangle_E\}_{\mathrm{sing}}=\tfrac{1}{2}\{\sum_{\mathbf{q}}J^{\mp}\langle\langle s^{\pm}c_{\mathbf{q}}|c^{+}_{\mathbf{k}'}\rangle\rangle_E\}_{\mathrm{sing}}$$

$$=\mp\tfrac{1}{2}J^{\mp}\{\sum_{\mathbf{p}\mathbf{q}\mathbf{r}}(E-\varepsilon_{\mathbf{q}}\mp\varDelta)^{-1}$$

$$\cdot\langle\langle c^{+}_{\mathbf{p}}c_{\mathbf{q}}c_{\mathbf{r}}(J^{\pm}s^{z}-J^{z}s^{\pm})|c^{+}_{\mathbf{k}'}\rangle\rangle_E\}_{\mathrm{sing}}$$

$$=\pm\tfrac{1}{2}(J^{\mp}J^{z}s^{\pm}-J^{+}J^{-}s^{z})\frac{\rho\,\xi}{E-\varepsilon_{\mathbf{k}'}}$$

Interaction with local excitations III

Equations

$$\frac{dJ_{ef}^{\mp}}{d\xi} = \pm\rho J_{ef}^{\mp} J_{ef}^{z}, \qquad \frac{dJ_{ef}^{\pm}}{d\xi} = 0,$$

$$\frac{dJ_{ef}^{z}}{d\xi} = \mp\tfrac{1}{2}\rho J_{ef}^{+} J_{ef}^{-}.$$

Always weak coupling regime!

Solution

$$(J_{ef}^{z})^2 + J_{ef}^{+} J_{ef}^{-} = J_0^2 = \mathrm{const} \qquad (J_0 > 0)$$

$$J_{ef}^{z} = J_0 \frac{\exp(\mp J_0 \rho \xi) - K}{\exp(\mp J_0 \rho \xi) + K} \to \mp J_0, \qquad K \equiv \frac{J_0 - J^z}{J_0 + J^z}$$

$$J_{ef}^{+} J_{ef}^{-} = \frac{4 J_0^2 K \exp(\mp J_0 \rho \xi)}{[\exp(\mp J_0 \rho \xi) + K]^2} \to 0$$

$$\ln\left|\frac{\Delta}{\varepsilon \mp \Delta}\right| \to \left|\frac{\varepsilon \mp \Delta}{\Delta}\right|^{J_0 \rho}$$

Logarithmic singularity weakens to the (positive) power-law

Interaction with local phonons

(the same paper)

$$H=\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}c_{\mathbf{k}}^{+}c_{\mathbf{k}}+\omega_0 b^{+}b+Q(x)c^{+}c,$$

$$x=(2M\omega_0)^{-1/2}(b^{+}+b)$$

$$Q(x)=\lambda(b^{+}+b)+\mu(b^{+}+b)^2+\ldots.$$

Dispersionless (local) phonon, anharmonic coupling

Perturbative (well-known) result: $\delta\Sigma^{(2)}(E)=\lambda^2\rho\ln\left|\frac{\varepsilon-\omega_0}{\varepsilon+\omega_0}\right|$

The same procedure: Green's finction average on electron but *not* on phonon operators (operator in phonon space)

Operator structure of T-matrix

$$T(E)=A(E)+L_{+}(E)b^{+}+L_{-}(E)b+M_{--}(E)b^2$$
$$+2M_{+-}(E)b^{+}b+M_{++}(E)(b^{+})^2+\ldots.$$

Interaction with local phonons II

The equations of motion give to order $\kappa^3 (b^- \equiv b)$

$$
\begin{aligned}
&(E-\varepsilon_k)\, G_{\mathbf{k}\mathbf{k}'}(E) \\
&= \delta_{\mathbf{k}\mathbf{k}'} + \langle\langle Q(x)\, c \,|\, c^+_{\mathbf{k}'} \rangle\rangle_E , \\
&\quad \{\langle\langle Q(x)\, c \,|\, c^+_{\mathbf{k}'} \rangle\rangle_E\}_{\text{sing}} = \lambda \{\sum_{\mathbf{q}} \langle\langle b^{\mp} c_{\mathbf{q}} \,|\, c^+_{\mathbf{k}'} \rangle\rangle_E\}_{\text{sing}} \\
&= \mp \lambda \{\sum_{\mathbf{q}} (E-\varepsilon_{\mathbf{q}} \mp \omega_0)^{-1} \\
&\quad \cdot \sum_{\mathbf{p}\mathbf{r}} \langle\langle c^+_{\mathbf{p}}\, c_{\mathbf{q}}\, c_{\mathbf{r}} [\lambda + 2\mu(b^+ + b)] \,|\, c^+_{\mathbf{k}'} \rangle\rangle_E\}_{\text{sing}} \\
&= \mp \{\sum_{\mathbf{q}} f_{\mathbf{q}} (E-\varepsilon_{\mathbf{q}} \mp \omega_0)^{-1} \\
&\quad \cdot [\lambda^2 + (b^+ + b)(2\mu\lambda + \lambda^3 \sum_{\mathbf{p}} [(E-\varepsilon_{\mathbf{p}})^{-1} \\
&\quad - (E-\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{p}})^{-1}])]\}_{\text{sing}} (E-\varepsilon_{\mathbf{k}'})^{-1} .
\end{aligned}
$$

Within this approximation μ is unrenormalized

$$
L_+(E) = L_-(E) = \lambda_{\text{ef}}(E) \equiv L(E)
$$

Interaction with local phonons III

$$\frac{\mathrm{d}L}{\mathrm{d}\xi} = \mp 2\rho L(L^2 R + \mu), \qquad \varepsilon \to \pm\omega_0$$

where

$$\xi = \ln|\omega_0/(\varepsilon \mp \omega_0)|$$

$$R = \mathrm{Re}\,\mathscr{R}, \qquad \mathscr{R} = \sum_{\mathbf{p}} (E_{\mathrm{F}} - \varepsilon_{\mathbf{p}} + i0)^{-1} \equiv R - i\pi\rho.$$

Solution: $$L^2(\xi) = \lambda^2 \mu [(\lambda^2 R + \mu)\exp(\pm 4\mu\rho\xi) - \lambda^2 R]^{-1}$$

We consider explicitly the case where $\varepsilon \to -\omega_0$ (the results for $\varepsilon \to \omega_0$ differ in the change $R \to -R$, $\mu \to -\mu$ only). According to (23) three types of effective interaction behaviour are possible

(i) $R<0$, $\mu \leqq 0$ or $R \geqq 0$, $\mu < -\lambda^2 R$. The effective interaction tends to zero:

$$L(E) \approx \left(\frac{\lambda^2 \mu}{\lambda^2 R + \mu}\right)^{1/2} \left|\frac{E - E_{\mathrm{F}} + \omega_0}{\omega_0}\right|^{-2\mu\rho}. \tag{25}$$

(ii) $R<0$, $\mu>0$. There exists a stable fixed point

$$L^* = (-\mu/R)^{1/2}. \tag{26}$$

Since $L^* \sim \kappa$ here we do not reach the strong coupling region either. This case is similar to the large number of scattering channels limit in the Kondo problem

(iii) $R>0$, $\mu > -\lambda^2 R$. The quantity L becomes infinite at the point $\xi = \ln(\omega_0/T^*)$, the energy

$$T^* = \omega_0 \exp\left(-\frac{1}{4\mu\rho}\ln\frac{\lambda^2 R + \mu}{\lambda^2 R}\right)$$
$$= \omega_0 \left(1 + \frac{\mu}{\lambda^2 R}\right)^{-1/4\mu\rho} \tag{27}$$

being the boundary of the strong coupling region. It plays the role of the Kondo temperature.

Interaction with local phonons IV

Conditions of strong-coupling regime

$$E \to E_F - \omega_0, \quad R > 0, \quad \mu > -\lambda^2 R$$

or at

$$E \to E_F + \omega_0, \quad R < 0, \quad \mu < \lambda^2 R.$$

Singularity on one side is much stronger than on the other (check by STM?!)

5. The Infrared Singularities in the Local Excitation Green's Function. The Orthogonality Catastrophe

$$D(\omega) = \langle\langle B | B^+ \rangle\rangle_\omega, \quad B = s^+, b.$$

$$D(\omega) = -E_F^{-\alpha}[E_0 - \omega - i\Gamma(\omega)]^{-1+\alpha}$$

$$\Gamma(\omega) = \frac{\pi}{4} J_\perp^2 \sum_{\mathbf{k}\mathbf{k}'} (f_{\mathbf{k}'} - f_{\mathbf{k}})\, \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \omega)$$

$$\approx \frac{\pi}{4} \omega J_\perp^2 \rho^2$$

$\alpha = (J^z \rho)^2$ For phonons $\alpha = 4(\lambda^2 R + \mu)^2 \rho^2$

Effective splitting

$$\Delta_{ef} = \Delta(\Delta/E_F)^\beta \qquad \beta = \tfrac{1}{2}(J_\perp \rho)^2$$

Ground-state energy

$$\delta \mathscr{E} = -\tfrac{1}{2} \Delta (\Delta/E_F)^\beta$$

Kondo effect and spin dynamics

Kondo effect, spin dynamics and magnetism in anomalous rare earth and actinide compounds

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Z. Phys. B – Condensed Matter 75, 67–76 (1989)

$$H = H_0 + H_{sf},\; H_0 = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^+_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + H_f$$

$$H_f = -\sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}} \mathbf{S}_{\mathbf{q}}$$

$$H_{sf} = -I \sum_{\mathbf{kq}\alpha\beta} \boldsymbol{\sigma}_{\alpha\beta} \mathbf{S}_{\mathbf{q}} c^+_{\mathbf{k}+\mathbf{q}\alpha} c_{\mathbf{k}\beta}$$

No need to add intersite exchange by hand,
it arises in the model (RKKY) – but convenient!

Electron Green's function – equation of motion (EOM) approach

$$\langle\langle c_{\mathbf{k}\uparrow} | c^+_{\mathbf{k}\uparrow} \rangle\rangle_E \equiv [E - \varepsilon_{\mathbf{k}} - \Sigma^{\uparrow}_{\mathbf{k}}(E)]^{-1} \quad (\mathrm{Im}\, E > 0)$$

the equations of motion

$$E\langle\langle A|B\rangle\rangle_E = \langle\{A, B\}\rangle + \langle\langle [A, H]|B\rangle\rangle_E,$$

$$E\langle\langle A|B\rangle\rangle_E = \langle\{A, B\}\rangle + \langle\langle A|[H, B]\rangle\rangle_E$$

we derive

$$\Sigma^{\uparrow}_{\mathbf{k}}(E) = I^2 \sum_{\mathbf{qp}\alpha\beta} \langle\langle \boldsymbol{\sigma}_{\uparrow\beta} \mathbf{S}_{-\mathbf{q}} c_{\mathbf{k}+\mathbf{q}\beta} | \boldsymbol{\sigma}_{\alpha\uparrow} \mathbf{S}_{\mathbf{p}} c^+_{\mathbf{k}+\mathbf{p}\alpha} \rangle\rangle^{\mathrm{irr}}_E$$

Kondo effect and spin dynamics II

Treating spin-dynamics *exactly*, representation of exact (multispin) eigenstates of H_f, $|n>$

$$S_{\mathbf{q}}^{\alpha} = \sum_{mn} (S_{\mathbf{q}}^{\alpha})_{mn} X^{nm},$$

$$X^{nm} = |n\rangle\langle m|, \; H_f |m\rangle = \varepsilon_m |m\rangle$$

To second order in I

$$\delta \sum\nolimits_{\mathbf{k}}^{(2)}(E) = I^2 \sum_{\substack{\mathbf{pq}\alpha\beta \\ mn}} \frac{\langle \{ (\boldsymbol{\sigma}_{\uparrow\beta} \mathbf{S}_{-\mathbf{q}})_{mn} X^{nm} c_{\mathbf{k}+\mathbf{q}\beta}, \boldsymbol{\sigma}_{\alpha\uparrow} \mathbf{S}_{\mathbf{p}} c^{+}_{\mathbf{k}+\mathbf{q}\alpha} \} \rangle}{E - \varepsilon_{\mathbf{k}+\mathbf{q}} + \varepsilon_n - \varepsilon_m}$$

Using the transformations

$$\frac{1}{\varepsilon + \varepsilon_n - \varepsilon_m + \mathrm{i}0} = - \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi \mathrm{i}} \times \frac{1}{\varepsilon - \omega + \mathrm{i}0} \, \frac{1}{\omega + \varepsilon_n - \varepsilon_m + \mathrm{i}0}$$

$$\sum_{mn} \frac{(S^{\alpha}_{-\mathbf{q}})_{mn} X^{nm}}{\omega + \varepsilon_n - \varepsilon_m} = \mathrm{i} \int_0^{\infty} \mathrm{d}t \, \mathrm{e}^{\mathrm{i}\omega t} S^{\alpha}_{-\mathbf{q}}(t)$$

Kondo effect and spin dynamics III

$$\delta\Sigma_{\mathbf{k}}^{(2)}(E) = 3I^2 \sum_{\mathbf{q}} \int_{-\infty}^{\infty} d\omega \mathscr{J}_{\mathbf{q}}(\omega) \times \left(\frac{1-f_{\mathbf{k}+\mathbf{q}}}{E-\varepsilon_{\mathbf{k}+\mathbf{q}}-\omega} + \frac{f_{\mathbf{k}+\mathbf{q}}}{E-\varepsilon_{\mathbf{k}+\mathbf{q}}+\omega} \right)$$

where $f_{\mathbf{q}} = f(\varepsilon_{\mathbf{q}})$ is the Fermi function,

$$\mathscr{J}_{\mathbf{q}}(\omega) = \sum_{mn} w_m |(S_{\mathbf{q}}^z)_{mn}|^2 \delta(\omega + \varepsilon_n - \varepsilon_m)$$

(w_m are the Gibbs factors) is the spectral density

$$\langle S_{-\mathbf{q}}^z(t) S_{\mathbf{q}}^z \rangle = \int_{-\infty}^{\infty} d\omega e^{i\omega t} \mathscr{J}_{\mathbf{q}}(\omega).$$

In PM phase, no "Kondo" (logarithmically divergent) contributions to the self-energy

Kondo contributions to the self-energy

$$\delta\Sigma_{\mathbf{k}}^{(3)}(E) = I^2 \sum_{\mathbf{p}\mathbf{q}\alpha\beta} \sum_{mn} (E - \varepsilon_{\mathbf{k}+\mathbf{q}} + \varepsilon_n - \varepsilon_m)^{-1}$$
$$\times \{ \langle c_{\mathbf{k}+\mathbf{p}\alpha}^{+} c_{\mathbf{k}+\mathbf{q}\beta} [\boldsymbol{\sigma}_{\alpha\uparrow}\mathbf{S}_{\mathbf{p}}, (\boldsymbol{\sigma}_{\uparrow\beta}\mathbf{S}_{-\mathbf{q}})_{mn} X^{nm}] \rangle$$
$$+ I f_{\mathbf{k}+\mathbf{q}} \sum_{m'n'} (E - \varepsilon_{\mathbf{k}+\mathbf{p}} + \varepsilon_{n'} - \varepsilon_{m'})^{-1}$$
$$\times \langle [(\boldsymbol{\sigma}_{\uparrow\beta}\mathbf{S}_{-\mathbf{q}})_{mn} X^{nm}, \boldsymbol{\sigma}_{\beta\alpha}\mathbf{S}_{\mathbf{q}-\mathbf{p}}]$$
$$\times (\boldsymbol{\sigma}_{\alpha\uparrow}\mathbf{S}_{\mathbf{p}})_{n'm'} X^{m'n'} \rangle \}.$$

The answer

$$\mathrm{Im}\,\delta\Sigma_{\mathbf{k}}^{(3)}(E) = 12\pi I^3 \rho \int_{-\infty}^{\infty} d\omega \sum_{\mathbf{q}} \frac{f_{\mathbf{k}+\mathbf{q}}}{E - \varepsilon_{\mathbf{k}+\mathbf{q}} - \omega} \mathscr{J}_{\mathbf{q}}(\omega)$$

Without spin-dynamics it gives exactly the original (Kondo) logarithmically divergent contribution in the third-order in I

Kondo effect and spin dynamics IV

Spin diffusion approximation $$\mathscr{I}_{\mathbf{q}}(\omega) = \frac{S(S+1)}{3\pi} \frac{Dq^2}{\omega^2 + (Dq^2)^2}$$

$$\delta\tau^{-1}(E) = 4\pi I^3 \rho^2 S(S+1) \ln \frac{E^2 + d^2}{W^2} \qquad d = 4Dk_F^2 \sim \bar{\omega}$$

Logarithmic singularity is smeared, that is, Kondo effect in suppressed

Renormalization of magnetic susceptibility

$$\chi = (S_0^z, S_0^z) = \int_0^\beta \mathrm{d}\lambda \langle \mathrm{e}^{\lambda H} S_0^z \mathrm{e}^{-\lambda H} S_0^z \rangle$$

Second-order in I

$$\chi = \frac{S(S+1)}{3T} + \chi^{(2)}, \quad \chi^{(2)} = \int_0^\beta \mathrm{d}\lambda \int_\lambda^\beta \mathrm{d}u_1 \int_0^\lambda \mathrm{d}u_2$$

$$\times \langle [S_0^z, \mathrm{e}^{u_1 H_0} H_{sf} \mathrm{e}^{-u_1 H_0}] S_0^z \mathrm{e}^{u_2 H_0} H_{sf} \mathrm{e}^{-u_2 H_0} \rangle$$

Kondo effect and spin dynamics V

$$\chi^{(2)}_{\mathrm{sing}} = \frac{4I^2}{T}\sum_{\mathbf{pq}}\int_{-\infty}^{\infty} d\omega\, \mathscr{J}_{\mathbf{p}-\mathbf{q}}(\omega)\frac{f_{\mathbf{p}}(1-f_{\mathbf{q}})}{(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{q}}+\omega)^2}$$

In spin-diffusion approximation $\chi^{(2)}_{\mathrm{sing}} \approx \frac{S(S+1)}{3T} 2I^2\rho^2 \ln\frac{T^2+d^2}{W^2}$

Back effect: renormalization of spin dynamics

One needs to calculate inhomogeneous susceptibility and come to the real space

$$J(\mathbf{r}, T) = J(\mathbf{r})\left[1 + 4I^2\rho^2(1-\alpha_{\mathbf{r}}^2)\ln\frac{W}{T}\right]$$

$$\alpha_{\mathbf{r}} = \langle e^{i\mathbf{kr}}\rangle_{\varepsilon_{\mathbf{k}} = E_F} = \frac{\sin k_F r}{k_F r}$$

Exchange integrals contain Kondo logarithms – one needs to do everything **self-consistently**

Scaling theory of Kondo lattices

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Scaling picture of magnetism formation in the anomalous f-electron systems: Interplay of the Kondo effect and spin dynamics

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(Received 27 December 1996; revised manuscript received 15 May 1997)

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Scaling theory of magnetic ordering in the Kondo lattices with anisotropic exchange interactions

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Non-Fermi-liquid behavior in Kondo lattices induced by peculiarities of magnetic ordering and spin dynamics

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Scaling theory of Kondo lattices II

$$H=\sum_{\mathbf{k}\sigma} t_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + H_f + H_{sf} = H_0 + H_{sf}$$

$$H_f=\sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}} \mathbf{S}_{\mathbf{q}}, \quad H_{sf}=-\sum_{\mathbf{k}\mathbf{k}'\alpha\beta} I_{\mathbf{k}\mathbf{k}'} \mathbf{S}_{\mathbf{k}-\mathbf{k}'} \boldsymbol{\sigma}_{\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}'\beta}$$

Without interaction of different spins and at $I < 0$
Kondo temperature (border of strong coupling)

$$T_K = D\exp(1/2I\rho)$$

Alternative model: SU(N) Coqblin - Schrieffer model

$$H_f=\frac{1}{2}\sum_{\mathbf{q}} J_{\mathbf{q}} \sum_{M,M'=-S}^{S} X^{MM'}_{-\mathbf{q}} X^{M'M}_{\mathbf{q}} \qquad H_{sf}=-I\sum_{\mathbf{k}\mathbf{k}'MM'} X^{MM'}_{\mathbf{k}'-\mathbf{k}} c^{\dagger}_{\mathbf{k}'M'} c_{\mathbf{k}M}$$

$$X_i^{MM'} = |iM\rangle\langle iM'|$$

Three phases considered: FM, AFM, and PM, different spin dynamics

Scaling theory of Kondo lattices III

PM phase: renormalization of effective coupling constant

Second-order expression in I for electron self-energy

$$\Sigma_{\mathbf{k}}^{(2)}(E) = I^2 P \sum_{\mathbf{q}} \frac{1}{E - t_{\mathbf{k}-\mathbf{q}}}$$

$$P = \begin{cases} [l]S(S+1), & \text{model } (2) \\ 1 - 1/N^2, & \text{model } (7). \end{cases}$$

Third-order expression with spin dynamics (for s-d model $N = 2$)

$$\Sigma_{\mathbf{k}}^{(3)}(E) = -I^3 PN \int_{-\infty}^{\infty} d\omega \sum_{\mathbf{q},\mathbf{p}} \mathcal{J}_{\mathbf{q}}(\omega) \frac{n_{\mathbf{k}-\mathbf{q}}}{E - t_{\mathbf{k}-\mathbf{q}} - \omega} \times \left(\frac{1}{E - t_{\mathbf{k}-\mathbf{p}}} - \frac{1}{t_{\mathbf{k}-\mathbf{q}} - t_{\mathbf{k}-\mathbf{p}}} \right),$$

parameter $I \rightarrow I_{ef} = I + \delta I_{ef}$ is determined by "including" $\mathrm{Im}\Sigma_{\mathbf{k}}^{(3)}(E)$ into $\mathrm{Im}\Sigma_{\mathbf{k}}^{(2)}(E)$, and is given by

$$\delta I_{ef} = -\frac{N}{2} I^2 \int_{-\infty}^{\infty} d\omega \sum_{\mathbf{q}} \mathcal{J}_{\mathbf{q}}(\omega) \frac{n_{\mathbf{k}-\mathbf{q}}}{E - t_{\mathbf{k}-\mathbf{q}} - \omega} \qquad \text{(B5)}$$

Scaling theory of Kondo lattices IV

PM phase: renormalization of magnetic moment

$$\chi = (S^z, S^z) \equiv \int_0^{1/T} d\lambda \langle \exp(\lambda H) S^z \exp(-\lambda H) S^z \rangle$$

Expanding to second order in I we derive (cf. Ref. 9)

$$\chi = \overline{S_{ef}^2}/3T, \quad \overline{S_{ef}^2} = S(S+1)[1-L],$$

$$L = 2RI^2 \int_{-\infty}^{\infty} d\omega \sum_{\mathbf{kq}} \mathcal{J}_{\mathbf{q}}(\omega) \frac{n_{\mathbf{k}}(1-n_{\mathbf{k-q}})}{(t_{\mathbf{k}} - t_{\mathbf{k-q}} - \omega)^2}$$

where we have introduced the notation

$$R = \begin{cases} [l], & \text{model (2)} \\ N/2, & \text{model (7)}. \end{cases}$$

Scaling theory of Kondo lattices V

PM phase: renormalization of spin dynamics

$$\omega_{\mathbf{q}}^2 = (\dot{S}^z_{-\mathbf{q}}, \dot{S}^z_{\mathbf{q}})/(S^z_{-\mathbf{q}}, S^z_{\mathbf{q}}).$$

To second order in I we derive (cf. Refs. 10 and 13)

$$(\omega_{\mathbf{q}}^2)_0 = \frac{4}{3} S(S+1) \sum_{\mathbf{p}} (J_{\mathbf{q}-\mathbf{p}} - J_{\mathbf{p}})^2,$$

$$\delta\omega_{\mathbf{q}}^2/\omega_{\mathbf{q}}^2 = (1-\widetilde{\alpha}_{\mathbf{q}})\,\delta\overline{S^2_{ef}}/\overline{S^2_{ef}} = -(1-\widetilde{\alpha}_{\mathbf{q}})L,$$

$$\widetilde{\alpha}_{\mathbf{q}} = \sum_{\mathbf{R}} J_{\mathbf{R}}^2 \left(\frac{\sin k_F R}{k_F R}\right)^2 [1-\cos\mathbf{qR}] / \sum_{\mathbf{R}} J_{\mathbf{R}}^2 [1-\cos\mathbf{qR}]$$

Scaling theory of Kondo lattices VI

Magnetically ordered phases: renormalization of effective coupling constant

$$\delta I_{ef} = -[\Sigma^{\mathrm{FM}}_{\mathbf{k}\uparrow}(E) - \Sigma^{\mathrm{FM}}_{\mathbf{k}\downarrow}(E)]/(2S[l])$$

$$\delta I_{ef} = -\Sigma^{\mathrm{AFM}}_{\mathbf{k},\mathbf{k}+\mathbf{Q}}(E)/(S[l])$$

Second order in I

$$\Sigma^{\mathrm{FM}}_{\mathbf{k}\uparrow}(E) = 2RI^2S\sum_{\mathbf{q}} \frac{n_{\mathbf{k}-\mathbf{q}}}{E - t_{\mathbf{k}-\mathbf{q}} + \omega^{\mathrm{FM}}_{\mathbf{q}}},$$

$$\Sigma^{\mathrm{FM}}_{\mathbf{k}\downarrow}(E) = 2RI^2S\sum_{\mathbf{q}} \frac{1 - n_{\mathbf{k}-\mathbf{q}}}{E - t_{\mathbf{k}-\mathbf{q}} - \omega^{\mathrm{FM}}_{\mathbf{q}}}.$$

$$\Sigma^{\mathrm{AFM}}_{\mathbf{k},\mathbf{k}+\mathbf{Q}}(E) = 2RI^2S\sum_{\mathbf{q}} \frac{n_{\mathbf{k}-\mathbf{q}}(E - t_{\mathbf{k}-\mathbf{q}})}{(E - t_{\mathbf{k}-\mathbf{q}})^2 - (\omega^{\mathrm{AFM}}_{\mathbf{q}})^2}$$

Important: in magnetically ordered phase “Kondo-like” logarithms arise already in the second order (in PM phase: only in third)

Scaling theory of Kondo lattices VII

RG equation for coupling constant (FM phase as an example)

$$\Sigma^{\mathrm{FM}}_{\mathbf{k}\uparrow}(E)=2RI^2S\sum_{\mathbf{q}}\frac{n_{\mathbf{k}-\mathbf{q}}}{E-t_{\mathbf{k}-\mathbf{q}}+\omega^{\mathrm{FM}}_{\mathbf{q}}},$$

Split the integration region over the layers

$$\Sigma^{\mathrm{FM}}_{\mathbf{k}\downarrow}(E)=2RI^2S\sum_{\mathbf{q}}\frac{1-n_{\mathbf{k}-\mathbf{q}}}{E-t_{\mathbf{k}-\mathbf{q}}-\omega^{\mathrm{FM}}_{\mathbf{q}}}.$$

$$C<t_{\mathbf{k}+\mathbf{q}}<C+\delta C$$

$$\delta I_{ef}(C)=I^2\sum_{C<t_{\mathbf{k}'}<C+\delta C}\left(\frac{1}{t_{\mathbf{k}'}+\omega_{\mathbf{k}'-\mathbf{k}}}+\frac{1}{t_{\mathbf{k}'}-\omega_{\mathbf{k}'-\mathbf{k}}}\right)$$

Averaging over Fermi surface

$$\delta I_{ef}(C)/\delta C=\rho^{-1}I^2\sum_{\mathbf{k},\mathbf{k}'}\delta(t_{\mathbf{k}})\,\delta(t_{\mathbf{k}'})\left(\frac{1}{C+\omega_{\mathbf{k}'-\mathbf{k}}}+\frac{1}{C-\omega_{\mathbf{k}'-\mathbf{k}}}\right).$$

“Debye” model for magnons

$$\omega_{\mathbf{q}}=D_sq^2$$

$$\delta I_{ef}(C)=\frac{\rho I^2}{\overline{\omega}}\delta C\ln\left|\frac{C-\overline{\omega}}{C+\overline{\omega}}\right|$$

$$\overline{\omega}=4D_sk_F^2$$

Scaling theory of Kondo lattices VIII

The most cumbersome part: calculation of spin Green functions and renormalization of spin frequencies and effective moments

The same trick: calculations of the contributions for specific layers of electron states

FM as example

$$\delta\omega_{\mathbf{q}}(C) = -2I^2S \sum_{\mathbf{k},\mathbf{k}',\, C<t_{\mathbf{k}}-t_{\mathbf{k}'}<C+\delta C} (J_{\mathbf{k}'-\mathbf{k}} + J_{\mathbf{q}} - J_{\mathbf{q}+\mathbf{k}-\mathbf{k}'} - J_0)\frac{1}{(t_{\mathbf{k}} - t_{\mathbf{k}'} - \omega_{\mathbf{k}'-\mathbf{k}})^2}$$

$$\delta\omega_{\mathbf{q}}(C)/\delta C = 2I^2S \sum_{\mathbf{k},\mathbf{k}'} \delta(t_{\mathbf{k}})\,\delta(t_{\mathbf{k}'})(J_{\mathbf{k}'-\mathbf{k}} + J_{\mathbf{q}} - J_{\mathbf{q}+\mathbf{k}-\mathbf{k}'} - J_0)\left(\frac{1}{C+\omega_{\mathbf{k}'-\mathbf{k}}} + \frac{1}{C-\omega_{\mathbf{k}'-\mathbf{k}}}\right)$$

$$\delta\,\overline{\omega_{ef}}(C) = 2\rho^2 I^2 (1-\alpha)\,\delta C \ln\left|\frac{C-\overline{\omega}}{C+\overline{\omega}}\right|$$

$$\delta\,\overline{S}_{ef}(C) = \frac{\rho^2 I^2 S}{\overline{\omega}}\,\delta C \ln\left|\frac{C-\overline{\omega}}{C+\overline{\omega}}\right|$$

$$\alpha_{\mathbf{q}} = \alpha = |\langle e^{i\mathbf{k}\mathbf{R}}\rangle_{t_{\mathbf{k}}=E_F}|^2 = \left(\frac{\sin k_F d}{k_F d}\right)^2$$

If we assume NN approximation for J it is constant!

Scaling theory of Kondo lattices IX

General RG equations (also added magnetic anisotropy)

$g_{ef}(C) = -2\rho I_{ef}(C)$ "exchange" spin-fluctuation energy $\overline{\omega}_{\text{ex}}(C)$

$\overline{S}_{\text{ef}}(C)$ gap in the spin-wave spectrum $\omega_0(C)$

$$\partial g_{ef}(C)/\partial C = \Lambda, \partial \ln \overline{S}_{\text{ef}}(C)/\partial C = -\Lambda/2,$$

$$\partial \ln \overline{\omega}_{\text{ex}}(C)/\partial C = -a\Lambda/2, \partial \ln \omega_0(C)/\partial C = -b\Lambda/2$$

$a = 1-\alpha$ for the paramagnetic (PM) phase, $a = 1-\alpha', b = 1$ for the antiferromagnetic (AFM) phase, $a = 2(1-\alpha''), b = 2$ for the ferromagnetic (FM) phase; $\alpha, \alpha', \alpha''$ are some averages over the Fermi surface (see Ref. 17); the quantity c is

$$\Lambda = \Lambda[C, \overline{\omega}_{\text{ex}}(C), \omega_0(C)]$$

$$= \frac{g^2_{\text{ef}}(C)}{|C|}\eta\left(\frac{\overline{\omega}_{\text{ex}}(C)}{|C|}, \frac{\omega_0(C)}{|C|}, \frac{\overline{\gamma}(C)}{|C|}\right)$$

$$\eta = \mathrm{Re}\int_{-\infty}^{\infty} d\omega \langle \mathcal{J}_{\mathbf{k}-\mathbf{k}'}(\omega)\rangle_{t_{\mathbf{k}}=t_{\mathbf{k}'}=0} \frac{1}{1-(\omega+i0)/C}$$

Scaling theory of Kondo lattices X

It follows from RG equations that we need to solve only one equation, other quantities follow from this solution

$$\overline{\omega}_{\text{ex}}(C)=\overline{\omega}_{\text{ex}}\exp(-a[g_{ef}(C)-g]/2),$$

$$\omega_0(C)=\omega_0\exp(-b[g_{ef}(C)-g]/2),$$

$$S_{ef}(C)=S\exp(-[g_{ef}(C)-g]/2),$$

Examples of the functions

$$\eta^{\text{PM}}(x)=x^{-1}\arctan x,$$

$$\eta^{\text{FM}}_{\uparrow,\downarrow}(x)=\pm x^{-1}\ln|1\pm x|,$$

$$\eta^{\text{AFM}}(x)=\begin{cases}-x^{-2}\ln|1-x^2|, & d=3\\ (1-x^2)^{-1/2}\theta(1-x^2), & d=2\end{cases}$$

(D = 3 for PM and FM)

Scaling theory of Kondo lattices XI

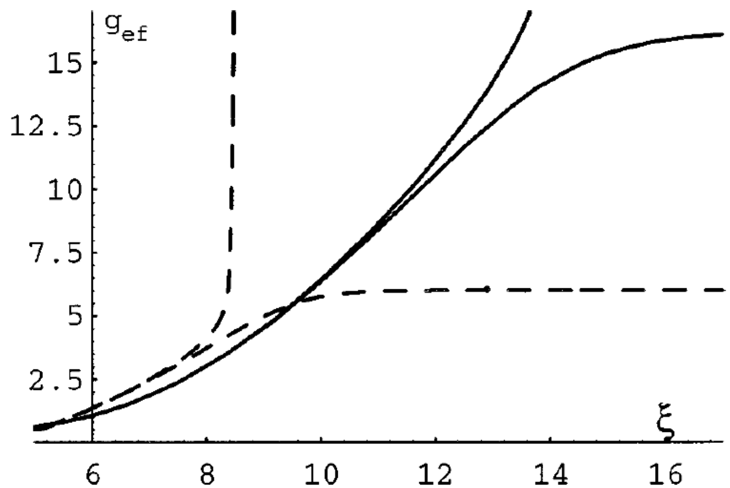

FIG. 2. The scaling trajectories $g_{ef}(\xi)$ in a paramagnet ($g=0.153\,92>g_c$, upper solid line, and $g=0.153\,91<g_c$ lower solid line) and a ferromagnet ($g=0.138\,68>g_c$, upper dashed line, and $g=0.138\,67<g_c$, lower dashed line) according to Eq. (62) with $N=2$, $\alpha=1/2$, $\lambda=5$, $\delta=1/100$.

$$\xi=\ln|D/C|, \quad \lambda=\ln(D/\overline{\omega})\gg 1$$

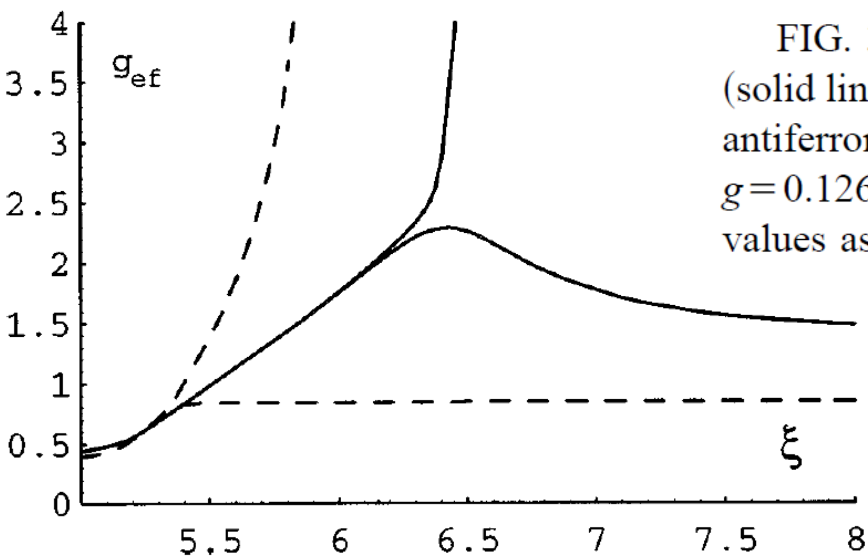

FIG. 3. The scaling trajectories $g_{ef}(\xi)$ in a 3D antiferromagnet (solid lines, $g=0.132\,038\,2>g_c$ and $g=0.132\,038\,1<g_c$) and a 2D antiferromagnet (dashed lines, $g=0.126\,671\,4>g_c$ and $g=0.126\,671\,3<g_c$) according to Eq. (62) with the same parameter values as in Fig. 2.

Scaling theory of Kondo lattices XII

Either at some cutoff coupling constant is divergent (Kondo regime with renormalized Kondo temperature) or remains always finite

$$T_K^* = D\exp(-\xi^*)$$

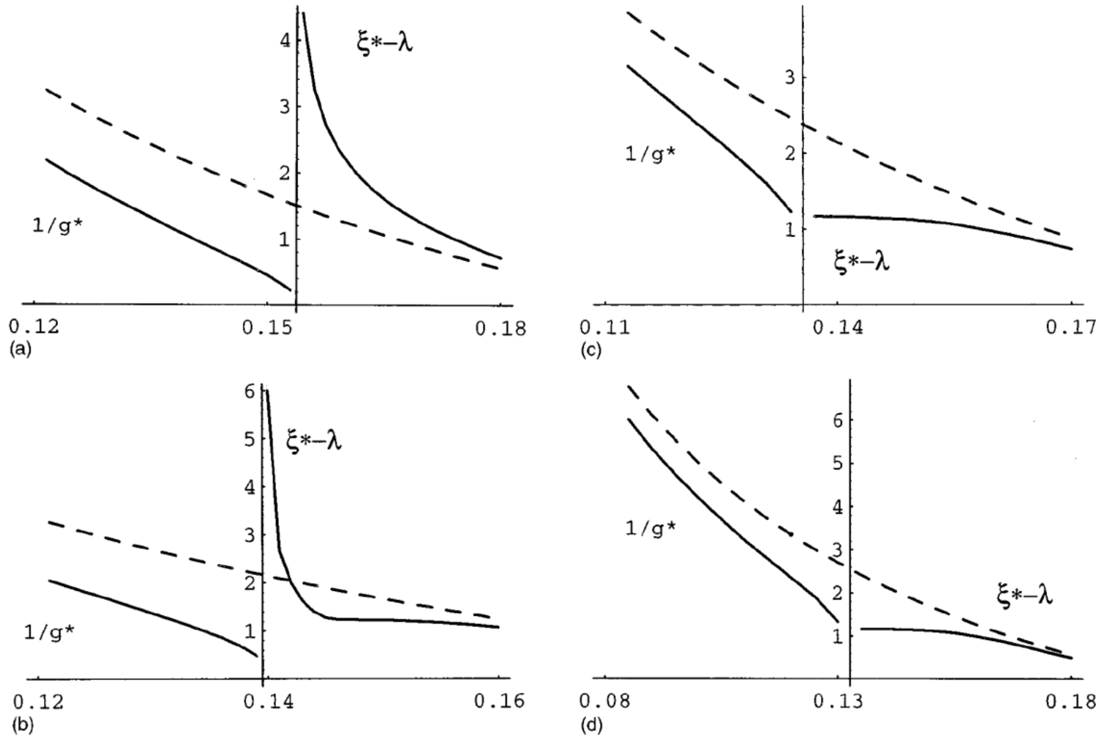

FIG. 5. The dependences $1/g^*(g)$ for $g<g_c$ and $\xi^*(g)-\lambda$ for $g>g_c$ in a paramagnet (a), ferromagnet (b), 3D antiferromagnet (c), and 2D antiferromagnet (d) according to Eq. (77). The dashed line is the curve $1/g-\lambda$, $\lambda=5$, $\alpha=1/2$, $N=2$. In the magnetically ordered phases we set $\delta=1/100$.

Conclusions

Anderson suggested a very simple and efficient trick to build RG equations for infrared divergences in solids (like Kondo problem, X-ray edge singularity, orthogonality catastrophe…)

It is not as general and as controllable as quantum-field RG but if it works this is the simplest way to the answer

Applicable not only to local problems!

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Suppressing backscattering of helical edge modes with a spin bath

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