From materials science to basic physics

Mikhail Katsnelson

1. Wave-particle duality in solids: itinerant vs localized behavior of electrons in correlated systems. How to understand ARPES data

2. Massless Dirac fermions in solids. Graphene as CERN on the desk: vacuum reconstruction, Klein paradox and all that
Instead of epigraph

You can get much further with a kind word and a gun than you can with a kind word alone (Al Capone)

You can get much further with an insight from experiment and mathematics than you can with mathematics alone
Microworld: waves are corpuscles, corpuscles are waves

Einstein, 1905 – for light (photons)
L. de Broglie, 1924 – electrons and other microparticles
Electrons are particles (you cannot see half of electron) but moves along all possible directions (interference)
Universal property of matter

Wave–particle duality of $C_{60}$ molecules

Markus Arndt, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw & Anton Zeilinger

Matter waves for $C_{60}$ molecules
From atoms to solids

Atoms:
- Electron-electron interaction plays crucial role
- Optimal filling
- Terms, multiplets
- Hund’s rules
- Making solids from atoms: electrons at given site

Crystals:
- Bloch waves propagating through the crystal
- Dispersion law
- Fermi surface
- Electron-electron Interaction just renormalizes parameters (Fermi liquid)...

How to combine?!

By S. Schubin and S. Wonsovsky.

Sverdlovsk Physical Technical Institute.

(Communicated by R. H. Fowler, F.R.S.—Received December 29, 1933.)

Proc. R. Soc. Lond. A 1934 145, published 2 June 1934

\[
\int \frac{e^2}{|x - x'|} \phi_\alpha^2(x) \phi_\alpha^2(x') \, dx \, dx' = A
\]

\[
\int \sum_{\gamma \neq \beta} \left[ G_\gamma(x) \phi_\alpha^2(x') + \frac{e^2}{|x - x'|} \phi_\gamma^2(x') \right] \phi_\alpha(x) \phi_\beta(x) \, dx \, dx' = L_{\alpha\beta}
\]

\[
\int \frac{e^2}{|x - x'|} \phi_\alpha^2(x) \phi_\beta^2(x') \, dx \, dx' = B_{\alpha\beta}
\]

\[
\int \frac{e^2}{|x - x'|} \phi_\alpha(x) \phi_\beta(x) \phi_\alpha(x') \phi_\beta(x') \, dx \, dx' = J_{\alpha\beta}
\]

Fig. 1.

Fig. 2.
The beginning: “Polar model” II

Schrödinger equation in “atomic representation” (double $f$, hole $g$, spin right $k$, spin left $h$)

$$\begin{align*}
&\{\varepsilon - s(A + D) - \left[ \sum_{f < f'} (B_{ff'} - J_{ff'}) + \sum_{g < g'} (B_{gg'} - J_{gg'}) - \sum_{f, g} (B_{f+} + J_{f+}) \right] C_{fgh} \\
&\quad + \sum_{h, k} J_{hk} [C (T_{hk} | fgh) - C (fg)] + \sum_{f, g} J_{rf} [C (T_{rg} | fgh) - C (fgh)] \\
&\quad + \sum_{f, p} L_{fp} C (T_{fp} | fgh) - \sum_{g, p} L_{gp} C (T_{gp} | fgh) = 0, \quad (9)
\end{align*}$$

Metal-insulator transition and Mott insulators

(II). The minimum energy corresponds to a certain $s = s_0$, where $0 < s_0 < n$. This case we have, for instance, when

$$A + 6(J - B) > 0, \quad A + 6J - 12L < 0.$$ 

Then, so long as $s$ remains small, the lowest energy level diminishes as $s$ increases; for a certain $s = s_0$ it attains a minimum and then again begins to increase. For such metals—at not very high temperatures—the number of “free” electrons approximates to twice this $s_0$ (electrons + holes!) and is therefore smaller than the number of atoms. In order to calculate $s_0$ in terms of our integrals, the energy must be evaluated up to the second approximation in powers of $s/n$; we shall not, however, make these rather cumbersome calculations here.

(III). The minimum energy corresponds to $s = 0$. This is the case when

$$A + 6(J - B) > 0, \quad A + 6J - 12L > 0.$$
When do we have a problem?

*sp* metals (empty or completely filled df shells): purely itinerant behavior

4f (rare earth) metals: *f* electrons are atomic like, *spd* electrons are itinerant

Photoemission (red) and inverse photoemission (green)

PES
Itinerant-electron magnets: coexistence

Local magnetic moments do exist above $T_C$ (Curie-Weiss law, spectroscopy, neutrons...)

$d$ electrons are itinerant (FS, chemical bonding, transport...)

Iron, majority spin FS

$4f$ electrons are normally pure localized but not $3d$
Correlated Electrons Step by Step: Itinerant-to-Localized Transition of Fe Impurities in Free-Electron Metal Hosts


Experiment: disappearance of multiplets

Calculations: increase of hybridization

Blue line: exact diagonalization for free atom
Dynamical Mean Field Theory

A.Georges, G.Kotliar, W.Krauth and M.Rozenberg, Rev. Mod. Phys. ‘96

A natural generalization of the familiar MFT to the problem of electrons in a lattice

Key idea: take one site out of a lattice and embed it in a self-consistent bath = mapping to an effective impurity problem

Effective impurity: atomic-like features, many-body problem

Putting into crystal: itinerant features, single-body problem
Combination with realistic calculations

A. Lichtenstein and MIK, PRB 57, 6884 (1998); JPCM 11, 1037 (1999)

V. Anisimov et al, JPCM 9, 7339 (1997)

PRB 57, 6884 (1998)

JPCM 11, 1037 (1999)
Ferromagnetism of transition metals: LDA+DMFT

Ferromagnetic Ni DMFT vs. LSDA:

- 30% band narrowing
- 50% spin-splitting reduction
- -6 eV satellite

LDA+DMFT with ME
J. Braun et al
PRL (2006)

Very good for Ni

Lichtenstein, MIK, Kotliar, PRL (2001)
Agreement is not bad (much better than LDA/GGA) but essentially worse than in nickel. Correlations in iron are not quite local.
ARPES for 3d metals

Effects of spin-dependent quasiparticle renormalization in Fe, Co, and Ni photoemission spectra: An experimental and theoretical study


Black – spin up, red – spin down
Upper panel – exper, lower - DMFT

Variation of U does not help too much for Fe

| TABLE I. Values of the experimental and theoretical mass enhancement factors $m^*/m_0$ for majority spin states at high symmetry points of the BBZ of Fe, Co, and Ni, respectively. The theoretical values are derived for $U(\text{Fe}) = 1.5 \text{ eV}$, $U(\text{Co}) = 2.5 \text{ eV}$, $U(\text{Ni}) = 2.8 \text{ eV}$. |
|---|---|---|---|
|   | Fe | Co | Ni |
| $\Gamma$ | 1.7 | 1.2 | 1.26 | 1.31 | 2.0 | 1.8 |
| $\Lambda$ | 1.1 | 1.2 | 1.29 | 1.31 | 1.9 | 1.8 |
How to go beyond DMFT: Dual fermion approach

A complicated change of variables in path integral such that DMFT becomes your new bare (noninteracting) Green’s function

Rubtsov, MIK, Lichtenstein 2008

The price: interaction action is now more ugly (but hopefully small)

Sometimes frogs are better

Very good zeroth-order approximation (exact in both weak and strong coupling limits); perturbation only in nonlocal correlations

Technically speaking: ladder diagram summation is good
Applications to single-band Hubbard model

Example: Fermi condensation at the Van Hove filling

Hope: to do multiband case and apply to Fe complicated but purely technical problem

Experiments with ultracold gases
The model: mapping on $a_{1g}$ band only (blue). Giant VHS – should be the main effect. Dual fermions in ladder approximation.

Physics: bound state (spin polaron) in FM $t$-$J$ model (MIK 1982)

Band splitting beyond DMFT
**5f electrons: localized vs itinerant**

Ratio of radius of f states to the half of interatomic distance in elemental solids

Magnetism of uranium compounds: Hill criterion

1- PM, 2- FM, 3 – AFM
(similar for Pu and Np compounds)

Localized-delocalized transition at $d \approx 3.5\text{Å}$
Elemental Pu

1. Six stable crystal phases at $p=0$
2. Negative thermal expansion in delta (fcc)-phase
3. Stabilization of delta’ phase on the way of Bain (fcc-bcc) deformation
4. Huge volume jumps at phase Transitions (very unusual for metals, the only other examples are alpha-gamma transition in Ce and “electronic” (s-d) transition in Cs at $p = 43$ kbar

Probably, the most interesting element in Periodic Table. Well... After carbon (and iron?!).
\[ \alpha - \delta \] transition in plutonium as a Mott transition in an f subsystem

M. I. Katsnel’son and I. V. Solov’ev
Institute of Metal Physics, Ural Branch of the Russian Academy of Sciences, 620219, Ekaterinburg, Russia

A. V. Trefilov
I. V. Kurchatov Institute of Atomic Energy, 123482, Moscow, Russia

(Submitted 31 July 1992)
Pis’ma Zh. Eksp. Teor. Fiz. 56, No. 5, 276–279 (10 September 1992)

Based on idea of atomic collapse and LDA:
Transition is not between Pu and Am but between different phases of Pu

Evidences of localized-delocalized transition: width of canonical f-bands and sensitivity of magnetism to the type of magnetic order
Local moments would help, but…
Is Pu magnetic?

Direct evidences from neutron scattering: no magnetic moments, neither ordered nor disordered, neither spin nor orbital

Hypothesis: basic $5f^6$ configuration like in Am ($S=3$, $L=3$, $J=L-S=0$)


Problems with the data of core-level spectroscopy and heat capacity
"Racah materials"

Calculations for δ-Pu vs PES

Peak at $E_f$ is not "Kondo" but due to $f^6$-$f^5$ multiplet transition

Three peak structure: $f^6$-$f^5$ multiplet transitions are better resolved than $f^5$-$f^4$ transitions.
We see multiplets $f^6 \rightarrow f^5$ but not $f^5 \rightarrow f^4$

Configuration is close to $f^5$ – good; nonmagnetic since mixed-valent – good; but many-body effects are due to the mixture of $f^6$ configuration

$$a_{i\lambda \sigma}^\dagger = (n + 1)^{1/2} \sum G_{S'L'}^{S'L'} C_{L\mu,lm}^{L',\mu'} C_{SM,\frac{1}{2}}^{S'M'} X_i(S'L'M'\mu',SLM\mu)$$

**fractional parentage coefficients**

$$X_i(\alpha\beta) = |i\alpha\rangle\langle i\beta|$$

**Hubbard X-operators**

$$X_i(\alpha\beta)X_i(\gamma\varepsilon) = \delta_{\beta\gamma}X_i(\alpha\varepsilon), \sum_{\alpha} X_i(\alpha\alpha) = 1$$
Why $\delta$-Pu is non-magnetic?

f-shell $n_f=5.2$: 80% $[f^5]$ + 20% $[f^6]$

Intermediate valence

Coulomb interaction between f- and d-states responsible for valence fluctuations

When there is a mixing of magnetic $[f^n]$ and non-magnetic $[f^{n+1}]$ multiplets the resulting state is non-magnetic.

Not a “theorem”, depends on hybridization parameters!
Mixed Valence Compounds

SmB$_6$

Narrow-gap semiconductor
Topological insulator
Nonmagnetic

LDA+DMFT results

We see multiplets $f^6 \rightarrow f^5$ but not $f^5 \rightarrow f^4$ – “Racah material”

Contrary to the case of TmSe
Part II: Graphene as CERN on the desk

Allotropes of Carbon

Diamond, Graphite

Graphene: prototype truly 2D crystal

Nanotubes

Fullerenes
Massless Dirac fermions

Pseudospin:
Sublattice A ↑
Sublattice B ↓

$sp^2$ hybridization, $\pi$ bands crossing
the neutrality point

Neglecting intervalley scattering:
massless Dirac fermions

Symmetry protected ($T$ and $I$)

FIG. 2: (color online) Band structure of a single graphene layer. Solid red lines are $\sigma$ bands and dotted blue lines are $\pi$ bands.
Massless Dirac fermions II

Spectrum near $K$ ($K'$) points is linear. Conical cross-points: provided by symmetry and thus robust property

Undoped  Electron  Hole
Massless Dirac fermions in condensed matter physics

1. d-wave superconductors
2. Vortices in superconductors and in superfluid helium-3
3. Topological insulators
4. Graphene

- Gap in high-Tc cuprates
- Electronic structure on surface of Bi$_2$Se$_3$
**Chiral tunneling and Klein paradox**

MIK, Novoselov, Geim, Nat. Phys. 2, 620 (2006)

Electronics: heterostructures (p-n-p junctions etc.)

(C) Florian Sterl
Klein paradox II

Ultrarelativistic

Nonrelativistic

Tunnel effect: momentum and coordinate are complementary variables, kinetic and potential energy are not measurable simultaneously.

Relativistic case: even the coordinate itself is not measurable, particle-antiparticle pair creation.
Klein paradox III

Transmission probability

Barrier width 100 \textit{nm}

Electron concentration outside barrier 0.5 \times 10^{12} \text{ cm}^{-2}

Hole concentration inside barrier 1 \times 10^{12} \text{ cm}^{-2} \text{ (red)} \text{ and } 3 \times 10^{12} \text{ cm}^{-2} \text{ (blue)}
Klein tunneling: Experimental confirmation

Evidence for Klein Tunneling in Graphene p-n Junctions

N. Stander, B. Huard, and D. Goldhaber-Gordon

Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 13 June 2008; published 16 January 2009)

Transport through potential barriers in graphene is investigated using a set of metallic gates capacitively coupled to graphene to modulate the potential landscape. When a gate-induced potential step is steep enough, disorder becomes less important and the resistance across the step is in quantitative agreement with predictions of Klein tunneling of Dirac fermions up to a small correction. We also perform magnetoresistance measurements at low magnetic fields and compare them to recent predictions.

Quantum interference and Klein tunnelling in graphene heterojunctions

Andrea F. Young and Philip Kim
Klein tunneling prevents localization

Back scattering is forbidden for chiral fermions! Magic angle = 0
Nonuniversal magic angle for bilayer exists!

Electrons cannot be locked by random potential relief neither for single-layer nor for bilayer graphene – absence of localization and minimal conductivity?!
Inhomogeneities are unavoidable

Freely suspended graphene membrane is corrugated


2D crystals in 3D space cannot be flat, due to bending instability

Atomistic simulations of intrinsic ripples

**Ripples and puddles**

Gibertini, Tomadin, Polini, Fasolino & MIK, PR B 81, 125437 (2010)

**FIG. 4.** (Color online) Top panel: fully self-consistent electronic density profile $\delta n(r)$ (in units of $10^{12}$ cm$^{-2}$) in a corrugated graphene sheet. The data reported in this figure have been obtained by setting $g_1=3$ eV, $\alpha_{ee}=0.9$ (this value of $\alpha_{ee}$ is the commonly used value for a graphene sheet on a SiO$_2$ substrate), and an average carrier density $\bar{n}_c=0.8 \times 10^{12}$ cm$^{-2}$. Bottom panel: same as in the top panel but for $\alpha_{ee}=2.2$ (this value of $\alpha_{ee}$ corresponds to suspended graphene).

**FIG. 9.** (Color online) One-dimensional plots of the self-consistent density profiles (as functions of $x$ in nm for $y=21.1$ nm) for different values of doping: $\bar{n}_c=0.8 \times 10^{12}$ cm$^{-2}$ (circles), $\bar{n}_c=3.96 \times 10^{12}$ cm$^{-2}$ (triangles), and $\bar{n}_c=3.17 \times 10^{13}$ cm$^{-2}$ (squares). The data reported in this figure have been obtained by setting $g_1=3$ eV and $\alpha_{ee}=2.2$. The inset shows $\delta n(r)$ (in units of $10^{12}$ cm$^{-2}$) at a given point $r$ in space as a function of the average carrier density $\bar{n}_c$ (in units of $10^{12}$ cm$^{-2}$).
Ripples and puddles II

Graphene on SiO$_2$

Gibertini, Tomadin, Guinea, MIK & Polini PR B 85, 201405 (2012)
Experimental STM data: V.Geringer et al (M.Morgenstern group)

FIG. 3: (Color online) Fully self-consistent induced carrier-density profile $\delta n(\mathbf{r})$ (in units of $10^{12}$ cm$^{-2}$) in the corrugated graphene sheet shown in Fig. 1. The data reported in this figure have been obtained by setting $g_1 = 3$ eV, $\alpha_{ee} = 0.9$, and an average carrier density $\bar{n}_c \approx 2.5 \times 10^{11}$ cm$^{-2}$. The thin solid lines are contour plots of the curvature $\nabla^2 h(\mathbf{r})$. Note that there is no simple correspondence between topographic out-of-plane corrugations and carrier-density inhomogeneity.
The role of Klein tunneling

Without Klein tunneling graphene near the neutrality point will be insulator and, anyway, will have a high enough mobility.

At the same time: a problem with transistors, one needs to use a complicated and indirect ways.

Crucial phenomenon for physics and electronic applications of graphene.

Field-Effect Tunneling Transistor Based on Vertical Graphene Heterostructures
L. Britnell, et al.
*Science* **335**, 947 (2012);

Tunneling transistor with vertical geometry.
Relativistic collapse for supercritical charges

Coulomb potential

\[ V_0(r) = \frac{Ze^2}{\epsilon r} \]

following Shytov, MIK & Levitov, PRL 99, 236801; 246803 (2007)

Naive arguments: Radius of atom \( R \), momentum \( \hbar/R \). Nonrelativistic case:

\[ E(R) \sim \frac{\hbar^2}{mR^2} - \frac{Ze^2}{R} \]

Minimum gives a size of atom.

Relativistic case: \( E(R) \sim \frac{\hbar c^*}{R} - \frac{Ze^2}{R} \)

Either no bound state or fall on the center.

Vacuum reconstruction at \( Z > 170 \)
**Supercritical charges II**

I. Pomeranchuk and Y. Smorodinsky, J. Phys. USSR 9, 97 (1945)

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**Superheavy nuclei**

Graphene:

\[ \nu \approx c/300, \quad \alpha_{eff} \approx 1 \]

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**FIG. 1:**

(a) Energy levels of superheavy atoms obtained from Dirac equation for Coulomb potential \(-Ze^2/r\), plotted as a function of \(\zeta = Ze\alpha\), where \(Z\) is nuclear charge, and \(\alpha = e^2/\hbar c\) is the fine structure constant. Energy is in the units of \(mc^2\).

(b) Energy levels for Coulomb potential regularized on the nuclear radius. As \(Z\) increases, the discrete levels approach the continuum of negative-energy states and dive into it one by one at supercritical \(Z > 170\) (from Ref.[23]).

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\(^{23}\) Y. B. Zeldovich and V. S. Popov, Usp. Fiz. Nauk 105, 403
Supercritical charges III

$$\beta = \frac{Ze^2}{\hbar v_F} \varepsilon > \frac{1}{2}$$

(A) Relativistic fall–down

M<Ze^2/c
\varepsilon>0

Klein tunneling

Quasi-local states

FIG. 3: (a) Local density of states (12) calculated at a fixed distance $\rho = 10^3 r_0$ from the charged impurity, where $r_0$ is a short-distance parameter of the order of carbon lattice spacing (from Ref.[10]). Peaks in the LDOS, which appear at supercritical $\beta$ and move to more negative energies at increasing $|\beta|$, correspond to the resonant states. (b) Spatial map of the density of states, shown for several values of $\beta$, with resonances marked by white arrows (from Ref.[11]). Note that the spatial width of the resonances decreases as they move to lower energies, $\Delta \rho \propto 1/|\varepsilon|$, while the linewidth increases, $\gamma \propto |\varepsilon|$. The oscillatory structure at positive energies represents standing waves with maxima at $k\rho \approx (n + \frac{1}{2})\pi$, similar to those studied in carbon nanotubes [26]. Energy is given in the units of $\varepsilon_0 = 10^{-3} \hbar \nu / r_0 \approx 30$ mV for $r_0 = 0.2$ nm.
Exper.: Tuning Z by Building Artificial Nuclei from Ca Dimers

Pseudomagnetic fields

Nearest-neighbour approximation: changes of hopping integrals

\[ \gamma = \gamma_0 + \left( \frac{\partial \gamma}{\partial \bar{u}_{ij}} \right)_0 \bar{u}_{ij} \]

\[ H = v_F \sigma \left( -i \hbar \nabla - \frac{e}{c} \mathbf{A} \right) \]

“Vector potentials”

\[ A_x = \frac{c}{2ev_F} (\gamma_2 + \gamma_3 - 2\gamma_1), \]
\[ A_y = \frac{\sqrt{3}c}{2ev_F} (\gamma_3 - \gamma_2), \]

\(K\) and \(K'\) points are shifted in opposite directions; Umklapp processes restore time-reversal symmetry

**Psedomagnetic fields II**

Within elasticity theory (continuum limit)

Shear deformations create vector potential

\[ A = \frac{\beta}{a} \begin{pmatrix} u_{xx} - u_{yy} \\ -2u_{xy} \end{pmatrix} \]

Dilatation creates scalar (electrostatic) potential

\[ V_1 = g_1(u_{xx} + u_{yy}) \]

\[ \beta = -\frac{\partial \ln t}{\partial \ln a} \approx 2 \]

**Pseudomagnetic field**

\[ B_S = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{1}{r} \frac{\partial A_r}{\partial \theta} - \frac{\partial A_\theta}{\partial r} - \frac{A_\theta}{r} \]
Gauge fields from mechanics: back to Maxwell

Electromagnetic fields as deformations in ether; gears and wheels

Zero-field QHE by strain engineering

F. Guinea, MIK & A. Geim, Nature Phys. 6, 30 (2010)

Can we create uniform (or almost uniform) pseudomagnetic field?

If you keep trigonal symmetry, quasi-uniform pseudomagnetic field can be easily created.

Normal stress applied to three edges size 1.4 μm, DOS in the center (0.5 μm)
Experimental confirmation

Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,1,2,* S. A. Burke,1,‡ K. L. Meaker,1 M. Panlasigui,1 A. Zettl,1,2 F. Guinea,3 A. H. Castro Neto,4 M. F. Crommie1,2‡

Fig. 2. (A) Sequence of eight $dI/dV$ spectra ($T \sim 7.5$ K, $V_{sd} = 20$ mV) taken in a line across a graphene nanobubble shown in the image in (B). Red lines are data with quartic background subtracted; black dotted lines are Lorentzian peak fits (center of peaks indicated by dots, with blue dots indicating $n = 0$). Vertical dash-dot lines follow the energy progression of each peak order. (C) Normalized peak energy versus $\text{sgn}(n) \sqrt{|n|}$ for peaks observed on five different nanobubbles follow expected scaling behavior from Eq. 1 (dashed line).

STM observation of pseudo-Landau levels

Graphene on Pt(111)
Combination of strain and electric field: Haldane insulator state

T. Low, F. Guinea & MIK, PRB 83, 195436 (2011)

Without inversion center combination of vector and scalar potential leads to gap opening

\[
\Delta = - \text{Tr} \left\{ \sigma_z \frac{2}{v_F} \int d^2 \vec{k} \frac{\text{Im} \left( \mathcal{V}_{\vec{k}} \right) \left( \vec{k} \vec{\sigma}, \left( \vec{A}_{\vec{k}} \vec{\sigma} \right) \right)}{|\vec{k}|^2} \right\}
\]

\[
\propto \int d^2 \vec{k} \frac{\text{Im}(\mathcal{V}_{\vec{k}}) \left( k_x A^y_{\vec{k}} - k_y A^x_{\vec{k}} \right)}{|\vec{k}|^2}
\]

(1)

Wrinkles plus modulated scalar potential at different angles to the wrinkling direction
Quantum pumping

Nanoelectromechanical resonator, periodic change of electric fields and pseudomagnetic fields (deformations) – a very efficient quantum pumping

Periodic electrostatic doping plus vertical deformation created pseudomagnetic vector potential

\[ V(t) = V_{dc} + V_{ac} \cos(\omega t) \]

\[ \mathcal{E}_{dg}(t) = \mathcal{E}_d \left(1 + \delta \mathcal{E}_d \sin(\omega t) \right)^{1/2} \]

\[ u_{xx}(t) = u_{xx} \left(1 + \delta u_{xx} \sin(\omega t + \phi) \right)^2 - \frac{\Delta L}{L} \]

\[ \mathcal{E}_d = h \nu f \left(\pi C_T V_{dc} / e \right)^{1/2} \]

\[ \delta \mathcal{E}_d = V_{ac} / V_{dc} \]

\[ u_{xx} = 8 h_0^2 / 3 L^2 \]

\[ \delta u_{xx} = a / h_0 \]
Quantum pumping II

Scattering problem

\[ \psi_f(x) = \begin{cases} 
\left( \frac{1}{\eta_f} \right) e^{i k_{xf} x} + R_v \left( -\eta_f^\dagger \right) e^{-i k_{xf} x} \\
\left( \frac{1}{\eta_g} \right) e^{i k_{xg} x} + \alpha_g \left( -\eta_g^\dagger \right) e^{-i k_{xg} x} \\
\mathcal{T}_v \sqrt{\frac{k_x k_f}{k_x k_f}} \left( \frac{1}{\eta_r} \right) e^{i k_{xr} x} 
\end{cases} \]

Pumping current

\[ I_v = i \frac{\epsilon \omega}{4 \pi^2} \sum_{k_y} \int_0^{2\pi/\omega} \int_{-\infty}^{\infty} \frac{d\epsilon}{\omega} \frac{\partial f_0(\epsilon)}{\partial \epsilon} \Omega_v(k_y, t) \]

\[ \Omega_v = \left( \frac{\partial T_v}{\partial t} \right) T_v^\dagger + \left( \frac{\partial R_v}{\partial t} \right) R_v^\dagger \]

Results

Asymmetric leads (different doping)
Collaborations

Sasha Lichtenstein and his group (Hamburg)
Olle Eriksson and his group (Uppsala)
Andre Geim, Kostya Novoselov and their group (Manchester)
Tim Wehling and his group (Bremen)

Sasha Shick (Prague), Alyosha Rubtsov (Moscow), Paco Guinea (Madrid), Leonya Levitov (MIT) and many others

And our group in Nijmegen